Question 1: Mention only one correction option.

i) The coordinate of an object is given as a function of time by $x = 7t - 3t^2$, where $x$ is in meters and $t$ is in seconds. Its average velocity over the interval from $t = 0$ to $t = 2$ s is:
A) 5 m/s
B) −5 m/s
C) −11 m/s
D) 1 m/s

ii) A car moving with an initial velocity of 25 m/s north has a constant acceleration of 3 m/s$^2$ south. After 6 seconds its velocity will be:
A) 7 m/s north
B) 7 m/s south
C) 43 m/s north
D) 20 m/s north

iii) A vector has a component of 10 m in the $+x$ direction, a component of 10 m in the $+y$ direction, and a component of 5 m in the $+z$ direction. The magnitude of this vector is:
A) 0 m
B) 15 m
C) 20 m
D) 25 m

iv) A vector in the $xy$ plane has a magnitude of 25 and an $x$ component of 12. The angle it makes with the positive $x$ axis is:
A) 26°
B) 29°
C) 61°
D) 64°

v) The standard 1-kg mass is attached to a compressed spring and the spring is released. If the mass initially has an acceleration of 5.6 m/s$^2$, the force of the spring has a magnitude of:
A) 2.8 N
B) 5.6 N
C) 11.2 N
D) 0 N

vi) An object attached to one end of a spring makes 20 complete vibrations in 10s. Its period is:
A) 2 Hz
B) 10 s
C) 0.5 Hz
D) 2 s
E) 0.50 s

vii) A wave is described by $y(x,t) = 0.1 \sin(3x - 10t)$, where $x$ is in meters, $y$ is in centimeters and $t$ is in seconds. The angular frequency is:
A) 0.10 rad/s
B) 3.0 rad/s
C) 10π rad/s
D) 20π rad/s
E) 10 rad/s

viii) The plot on right side shows a mass oscillating as $x = x_m \cos(\omega t + \phi)$. What are $x_m$ and $\phi$?
A) 1 m, 0°
B) 2 m, 0°
C) 4 m, 0°
D) 2 m, 90°
E) 4 m, 0°
ix) The displacement of a string is given by \( y(x,t) = y_m \sin(kx + \omega t) \). The wavelength of the wave is:

A) \( 2\pi/k \)  
B) \( k/\omega \)  
C) \( \omega k \)  
D) \( 2\pi/k \)

x) A 5.0-C charge is 10 m from a –2.0-C charge. The electrostatic force is on the positive charge is:

A) \( 9.0 \times 10^8 \) N toward the negative charge  
B) \( 9.0 \times 10^8 \) N away from the negative charge  
C) \( 9.0 \times 10^9 \) N toward the negative charge

xi) Two identical charges, 2.0 m apart, exert forces of magnitude 4.0 N on each other. The value of either charge is:

A) \( 1.8 \times 10^{-9} \) C  
B) \( 2.1 \times 10^{-5} \) C  
C) \( 4.2 \times 10^{-5} \) C  
D) \( 1.9 \times 10^5 \) C

xii) The electric field at a distance of 10 cm from an isolated point particle with a charge of \( 2 \times 10^{-9} \) C is:

A) 1.8 N/C  
B) 18 N/C  
C) 1800 N/C

xiii) An electric dipole consists of a particle with a charge of \( +6 \times 10^{-6} \) C at the origin and a particle with a charge of \( -6 \times 10^{-6} \) C on the x axis at \( x = 3 \times 10^{-3} \) m. Its dipole moment is:

A) \( 1.8 \times 10^{-8} \) C•m, in the positive x direction  
B) \( 1.8 \times 10^{-8} \) C•m, in the negative x direction  
C) 0 C•m, because the net charge is 0

xiv) A 10-ohm resistor has a constant current. If 1200 C of charge flow through it in 4 minutes what is the value of the current?

A) 3.0 A  
B) 5.0 A  
C) 20 A  
D) 120 A

xv) The figure on right side shows a junction. What is true of the currents?

A) \( i_1 = i_0 + i_2 \)  
B) \( i_2 = i_0 + i_1 \)  
C) \( i_1 = i_0 - i_2 \)  
D) \( i_1 = i_0 - i_1 \)

xvi) A wire has an electric field of 6.2 V/m and carries a current density of \( 2.4 \times 10^8 \) A/m². What is its resistivity?

A) \( 6.7 \times 10^{-10} \) Ω•m  
B) \( 1.5 \times 10^{-8} \) Ω•m  
C) \( 2.6 \times 10^{-8} \) Ω•m

xvii) An electron (charge = \( -1.6 \times 10^{-19} \) C) is moving at \( 3.0 \times 10^5 \) m/s in the positive x direction. A magnetic field of 0.80 T is in the positive z direction. The magnetic force on the electron is:

A) 0 N  
B) \( 4.5 \times 10^{-14} \) N in the positive z direction  
C) \( 4.5 \times 10^{-14} \) N in the negative z direction  
D) \( 4.5 \times 10^{-14} \) N in the positive y direction

xviii) The direction of magnetic field in a certain region of space is determined by firing a test charge into the region with its velocity in various directions in different trials. The field direction is:

A) one of the directions of the velocity when the magnetic force is zero  
B) the direction of the velocity when the magnetic force is a maximum  
C) the direction of the magnetic force

xix) Lines of the magnetic field produced by a long straight wire carrying a current are:

A) in the direction of the current  
B) opposite to the direction of the current  
C) leave the wire radially  
D) circles concentric with the wire

xx) Which one of the following parameters is not used to determine the magnetic force on a current-carrying wire in a magnetic field?

a) length of the wire  
b) radius of the wire
Question 2(a). Suppose that

\[ A = i \cos(\omega t) + j \sin(\omega t) \]

Where \( \omega \) is a constant. Find \( dA/dt \) (note that \( i \) and \( j \) behave as constants in differentiation). Show that \( dA/dt \) is perpendicular to \( A \). 

\[ \frac{dA}{dt} = -\omega \sin(\omega t) \hat{i} + j \omega \cos(\omega t) \hat{j} \]

\[ A \cdot \frac{dA}{dt} = -\omega \sin(\omega t) \cos(\omega t) + \omega \cos(\omega t) \sin(\omega t) = 0 \]

Question 2(b): A golfer claims that a golf ball launched with an elevation angle of 12° can reach a horizontal range of 250 m. Ignoring air friction, what would the initial speed of such a golf ball have to be? What maximum height would it reach?

\[ R = \frac{v_0^2 \sin(2\theta)}{g} \Rightarrow v_0 = \frac{77.7 \text{ m/s}}{9} \]

\[ H_{\text{max}} = \frac{v_0^2 \sin^2 \theta}{2g} = 13.3 \text{ m} \]
Question 2(c): A woman pushes horizontally on a wooden box of mass 60 kg sitting on a frictionless ramp inclined at an angle of 30° (see Figure below).

(i) Draw the “free-body” diagram for the box. (2 marks)
(ii) Calculate the magnitudes of all the forces acting on the box under the assumption that the box is at rest or in uniform motion along the ramp. (3 marks)

\[ F = 339.5 \, N \]
\[ N = 679 \, N \]

Figure for Question 2(c)

Question 3(a): Derive an expression for the time-period of simple pendulum. (5 marks)

See Textbook.

\[ T = 2\pi \sqrt{\frac{l}{g}} \]
Question 3(b) Explain in simple words why the time-period of simple pendulum is independent of its mass. (2 marks)

\[ \text{Changes mass} \iff \text{changes restoring force} \]
\[ \implies \text{constant acceleration} \]

Question 3(c): The equation of a wave travelling in the \(+x\) direction is given as
\[ y = y_m \sin(kx - \omega t + \phi) \]
Explain each term in the above equation and write mathematical expressions which show the relationship between “angular frequency and time period” and “wavelength and angular wave number”. (4 marks)

\[ a \]

\[ b \]
\[ v = \omega k \quad ; \quad T = \frac{2\pi}{k} \quad , \quad \omega = \frac{2\pi}{T} \quad , \quad \omega = 2\pi f \]

Question 4(a): Four charges of same magnitude are kept at 4 vertices of a square as shown in the figure below. (3 marks)

(i) Find the electric field at the center of the square i.e., at point P?
(ii) If I place a +ve charge +Q at point P, what will be the magnitude and direction of the force applied on charge +Q? What if I place a -ve charge -Q instead of +Q? (1+1=2 marks)

\[ i \quad \text{Zero} \]

\[ ii \quad \text{Zero, Zero} \]

Figure for Question 4(a)
Question 4(b): Suppose you have a solid sphere of radius $R$ with uniform volume charge density $\rho$. Find the electric field outside and inside the sphere.

\[ D = \frac{Q}{\frac{4}{3} \pi R^3} \Rightarrow Q = \frac{3}{4} \rho \pi R^3 \]

\[ E = \frac{1}{4} \frac{Q}{\pi R^2} \quad (R > r) \]

\[ E = \frac{1}{4} \frac{Q}{R^3} \quad (r < R) \]

Question 5(a): A parallel-plate capacitor consists of two strips of aluminum foil, each with an area of 0.20 m$^2$, separated by a distance of 0.10 mm. The space between the foils is empty. The two strips are connected to the terminals of a battery, which produces a potential difference of 200 volts between them. What is the capacitance of this capacitor? What is the electric charge on each plate? What is the strength of the electric field between the plates?

\[ (\text{i}) \quad C = \frac{\varepsilon_0 A}{d} = 0.018 \text{ mF} \]

\[ (\text{ii}) \quad Q = C \Delta V = 3.6 \times 10^{-6} \text{ C} \]

\[ (\text{iii}) \quad E = \frac{\Delta V}{d} = 2 \times 10^6 \text{ V/m} \]
Question 5(b): Six identical capacitors of capacitance \( C \) are connected as shown in Figure below. What is the net capacitance of the combination?

\[
C = \left( \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \right) ^{-1} + \left( \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \right) ^{-1} \\
C = \frac{2C}{3}
\]

Figure for Question 5(b)

Question 6(a): A current begins flowing into an initially uncharged capacitor at \( t = 0 \) and decreases to zero during \( 0 \leq t \leq 2.0 \text{ s} \) with a time dependence given by \( I = A \times (t - 2.0 \text{ s})^2 \), where \( A = 0.25 \text{ C/s}^2 \). What is the initial current? The current at \( 1.0 \text{ s} \)? How much charge is on the capacitor when the current stops at \( t = 2.0 \text{ s} \)?

\[
I(t = 0 \text{ s}) = 1 \text{ A} \\
I(t = 18) = 0.25 \text{ A} \\
Q = \int_0^2 I(t) \, dt = \frac{2}{3} \text{ Coulomb} = 0.67 \text{ C}
\]

Question 6(b): The magnitude \( J \) of the current density in a certain lab wire with a circular cross section of radius \( R = 2.00 \text{ mm} \) is given by \( J = (3.00 \times 10^8) r^2 \), with \( J \) in amperes per square meter and radial distance \( r \) in meters. What is the current through the outer section bounded by \( r = 0.900 \text{ R} \) and \( r = R \)?

\[
i = \int_{0.9R}^{R} J \, dA = \frac{1}{2} \pi R^2 \left( R^4 - 0.65 \pi R^4 \right)
\]

\[
R = 2.00 \text{ mm} = 0.002 \text{ m} \\
i = 2.59 \times 10^{-3} \text{ A}
\]
Question 6(c): Suppose a current $i$ is flowing through a uniform cylindrical conducting wire. Derive an expression of drift velocity in terms of $i$, $A$, $n$ and $e$. Here, $i$ is the current, $A$ is area of cross-section of cylindrical wire, $n$ is number of charge carriers per unit volume and $e$ is the charge on an electron.  

$$q = Ne$$
$$n = \frac{N}{AL} \Rightarrow N = nAL$$

$$q = Ne = nALe$$

$$t = \frac{L}{Vd}$$

$$i = \frac{q}{t} = nAeVd$$

$$Vd = \frac{i}{Ae}$$

$$Nd = \frac{i}{Ane}$$

Question 7(a): Derive an expression of angular frequency of a charged particle rotating in a uniform magnetic field.

$$qvB = \frac{mv^2}{r}$$

$$\omega = \frac{qvB}{m}$$

$$f = \frac{qvB}{2\pi m}$$

$$T = \frac{2\pi r}{V}$$

Question 7(b): A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m. 

$$ILB\sin\phi = mg$$

$$B = 1.6 \times 10^{-2} T$$
Question 8(a): The long dimension of the rectangular loop in Figure is \(400 \times 10^4\) m, and the magnetic field strength near loop has constant magnitude of \(2\) mT. Use mathematical form of the Ampere’s law to estimate the total current encircled by the rectangle.

\[\oint_B \cdot dB = 2 \int_0 \text{contribution} \]

\[\int_B \cdot dB = B \oint dr = B \oint_2 \]

For both sides:

\[I = \frac{2BL}{\mu_0} \]

\[I = 10^{12} A\]

Figure for Question 8(a)

Question 8(b): Derive an expression of magnetic field of a solenoid.

\[\oint B \cdot dl = B h + 0 + 0 + 0\]

\[B h = \mu_0 n i h\]

\[B = \mu_0 n i\]