1. In a circuit shown below, find the following:
   (i) Equivalent capacitance between $A$ and $D$.
   (ii) Charge on $12 \mu F$ capacitor.

![Figure 1: Capacitors in a combination configuration.](image)
Solution

(i)
\[ C_{AB} = 10 \text{\mu F} \]
\[ C_{BC} = \left( \frac{8 \times 8}{8 + 8} \right) + (8) + (8) = 4 + 8 + 8 = 20 \text{\mu F} \]
\[ C_{AC} = \frac{C_{AB} \times C_{BC}}{C_{AB} + C_{BC}} = \frac{10 \times 20}{10 + 20} = \frac{20}{3} \text{\mu F} \]
\[ C_{CD} = 8 + 12 = 20 \text{\mu F} \]
\[ C_{AD} = \frac{(20/3) \times 20}{(20/3) + 20} = 5 \text{\mu F} \]

(ii) Total charge, \[ q = C_{AD} \times V = (5 \times 10^{-6}) \times 400 = 0.002 \text{C} \]
The total charge will divide between the parallel capacitors connected in the branch CD.

P.D. between \( C \) and \( D \), \[ V_{CD} = \frac{q}{C_{CD}} = \frac{0.002}{20 \times 10^{-6}} = 100 \text{V} \]
\[ \Rightarrow \text{ Charge on 12 \mu F capacitor} = (12 \times 10^{-6}) \times 100 = 1.2 \times 10^{-3} \text{C} = 1.2 \text{\mu C} \]

2. A capacitor is filled with two dielectrics of the same dimensions but of dielectric constants \( \kappa_1 \) and \( \kappa_2 \) respectively. Find the capacitances in two possible arrangements as shown below:

![Figure 2: Dielectric with possible configuration.](image)

Solution:
(i) The arrangement shown in (i) is equivalent to two capacitors in series, each with plate area $A$ and plate separation $d/2$ i.e.,

$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} = \frac{2K_1 \varepsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \varepsilon_0 A}{d/2} = \frac{2K_2 \varepsilon_0 A}{d}$$

The equivalent capacitance $C'$ is given by;

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \varepsilon_0 A} + \frac{d}{2K_2 \varepsilon_0 A} = \frac{d}{2 \varepsilon_0 A} \left( \frac{1}{K_1} + \frac{1}{K_2} \right)$$

$$\therefore \quad C' = \frac{2 \varepsilon_0 A}{d} \left( \frac{K_1 K_2}{K_1 + K_2} \right)$$

(ii) The arrangement shown in (ii) is equivalent to two capacitors in parallel, each with plate area $A/2$ and plate separation $d$ i.e.,

$$C_1 = \frac{K_1 \varepsilon_0 (A/2)}{d} = \frac{K_1 \varepsilon_0 A}{2d}$$

$$C_2 = \frac{K_2 \varepsilon_0 (A/2)}{d} = \frac{K_2 \varepsilon_0 A}{2d}$$

The equivalent capacitance $C''$ is given by;

$$C'' = C_1 + C_2 = \frac{K_1 \varepsilon_0 A}{2d} + \frac{K_2 \varepsilon_0 A}{2d} = \frac{\varepsilon_0 A}{2d} (K_1 + K_2)$$

$$\therefore \quad C'' = \frac{\varepsilon_0 A}{2d} (K_1 + K_2)$$

3. A 20 m length of a certain wire has a resistance of $15 \, \Omega$. What length of this wire would have a resistance of $8 \, \Omega$? [5]

4. A platinum wire 80 cm long is to have a resistance of $0.10 \, \Omega$. What should its diameter be? The resistivity of platinum is $1.10 \times 10^{-7} \, \Omega \cdot \text{m}$. [5]