Question 1 (a). The equation of a transverse wave traveling along a string is

\[ y = (2.0\, \text{mm}) \sin[(20\, \text{m}^{-1}) \, x - (600\, \text{s}^{-1}) \, t]. \]

Find (a) amplitude, (b) frequency, (c) velocity (including sign) and (d) wavelength of the wave. Also find the maximum transverse speed of a particle in the string.

\(2+2+2+2+2=10\)

(a) The amplitude is \[ y_m = 2.0 \, \text{mm} \]

(b) \( w = 600 \, \text{rad/s} \quad ; \quad f = \frac{w}{2\pi} = \frac{600}{2\pi} \approx 95 \, \text{Hz} \)

(c) \( \omega = 2\omega \quad ; \quad v = \omega \, R = 600/20 = 30 \, \text{m/s} \quad \text{in x direction} \)

(d) \( \text{(wavelength)} \quad \lambda = \frac{2\pi}{\frac{\omega}{R}} = \frac{2\pi}{20} \, \text{m} = 0.31 \, \text{m} \)

(e) \[ \dot{y} = y_m \, \sin(kx - \omega t) \]

\[ \dot{y} = -\omega y_m \, \sin(kx - \omega t) \]

\[ \dot{y}_{\text{max}} = \omega y_m = (600)(2.0) \, \text{mm/s} = 1200 \, \text{mm/s} = 1.2 \, \text{m/s} \]
**Question 1 (b):** Calculate the expression for the time period of torsion pendulum. Also draw the diagram for a torsion pendulum.

For small twists the restoring torque in the wire is proportional to angular displacement (Hooke’s Law), so that

\[ T = -K \theta \]

\( K \) is torsional constant and depends on properties of wire.

The equation of motion for such a system can be written using angular form of Newton's 2nd Law

\[ \sum T = I \frac{d^2 \theta}{dt^2} \]

where \( I \) is rotational inertia of disk

We can write:

\[ -K \theta = I \frac{d^2 \theta}{dt^2} \]

or

\[ \frac{d^2 \theta}{dt^2} = \left( \frac{K}{I} \right) \theta \quad (1) \]

Equation (1) is similar to equation of motion of Simple Harmonic Oscillator (SHO) with a variable \( \theta \) with solution \( \theta = \theta m \cos (\omega t + \phi) \)

\[ \omega^2 = \frac{K}{I} \Rightarrow T = \frac{2 \pi \sqrt{I}}{K} \]

\( I = \frac{1}{2} MR^2 \) for disk with rotational axis in middle of disk.

**Question 1 (c):** A particle rotates counterclockwise in a circle of radius 3.00 m with a constant angular speed of 8.00 rad/s. At \( t = 0 \), the particle has an \( x \) coordinate of 2.00 m and is moving to the right. Determine the \( x \) coordinate as a function of time.

\( \text{(Amplitude of particle motion)} \quad A = \text{Radius of circle} = 3 \text{m.} \]
\( \omega = 8 \text{ rad/s} \)

We have

\[ x = A \cos (\omega t + \phi) \]
\[ x = 3 \cos (8t + \phi) \]

\[ x = 2 \text{ m at } t = 0 \]
\[ 2 = 3 \cos (\phi) \]
\[ \cos (\phi) = \frac{2}{3} \]
\[ \phi = \pm 0.841 \text{ radians} \]

For a positive \( \phi \), the particle is moving to the right (i.e., \( \omega > 0 \) and directed to the right)

\[ x = -A \omega \sin (\omega t + \phi) \]

At \( t = 0 \), particle is moving to right \( \omega t + \phi > 0 \) (the direction)

\[ x = -A \omega \sin \phi > 0 \]

\( -A \omega \sin \phi > 0 \) only if \( \phi \) is \(-\)ve

So, \( \phi \) should be \(-\)ve

\[ \phi = -0.841 \text{ radians} \]

The line of oscillation b/w \( OA \), or \( OB \) in angular amplitude

Oscillation takes place in \( x \)-plane

2-axis is along wire!
Question 2 (a): Consider three point charges located at the corners of a right triangle as shown in Figure 1, where \( q_1 = q_3 = 5.0 \, \mu\text{C}, q_2 = 2.0 \, \mu\text{C}, \) and \( a = 0.10 \, \text{m}. \) Find the resultant force exerted on \( q_3 \) in unit vector notation.

Also discuss what happens if the signs of all the charges were changed to the opposite signs. How would this affect the result for the resultant force.

\[
\begin{align*}
\theta &= \tan^{-1}\left(\frac{b}{a}\right) = 45^\circ \\
|\vec{F}_{13}| &= \frac{k|q_1||q_3|}{r^2} \\
&= \frac{9 \times 10^9 \left(5 \times 10^{-6}\right)\left(5 \times 10^{-6}\right)}{\left(0.1\right)^2} \quad \gamma = \frac{b}{a} \\
|\vec{F}_{13}| &= 11 \text{ N} \\
|\vec{F}_{23}| &= \frac{k|q_2||q_3|}{r^2} \\
&= \frac{9 \times 10^9 \left(2 \times 10^{-6}\right)\left(5 \times 10^{-6}\right)}{\left(0.1\right)^2} \\
|\vec{F}_{23}| &= 9.0 \text{ N} \\
\vec{F}_{13} &= -(9.0 \text{ N}) \hat{e}_x \\
\vec{F}_{23} &= (11 \text{ N}) \cos(45^\circ) \hat{e}_x + (11 \text{ N}) \sin(45^\circ) \hat{e}_y \\
\vec{F}_3 &= (7.9 \text{ N}) \hat{e}_x + (7.9 \text{ N}) \hat{e}_y \\
\vec{F} &= \vec{F}_{13} + \vec{F}_{23} \\
&= -(9.0 \text{ N}) \hat{e}_x + (7.9 \text{ N}) \hat{e}_x + (7.9 \text{ N}) \hat{e}_y \\
\vec{F} &= -(1.1 \text{ N}) \hat{e}_x + (7.9 \text{ N}) \hat{e}_y
\end{align*}
\]

What if sign of all three charges were changed to opposite signs?

The charge \( q_3 \) would still be attracted towards \( q_2 \) and repelled from \( q_1 \) with forces of same magnitude.

Thus the final result \( \vec{F} \) would be exactly the same.
**Question #2(c):** Two point charges $q_1 = +3 \mu C$ and $q_2 = -3 \mu C$ are located 20cm apart in vacuum. What is the electric field at the mid-point $O$ of the line $AB$ joining the two charges?

![Diagram of two charges with electric fields](image)

Electric field at point $O$ due to $3 \mu C$ charge:

$$E_1 = \frac{k|q_1|}{r^2} \hat{e}_x = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2} \hat{e}_x = (2.7 \times 10^6 \text{ N/C}) \hat{e}_x$$

Electric field at point $O$ due to $-3 \mu C$ charge:

$$E_2 = \frac{k|q_2|}{r^2} \hat{e}_x = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2} \hat{e}_x = (2.7 \times 10^6 \text{ N/C}) \hat{e}_x$$

Electric field at point $O$:

$$E = E_1 + E_2 = (2.7 \times 10^6 \text{ N/C}) \hat{e}_x + (2.7 \times 10^6 \text{ N/C}) \hat{e}_x = (5.4 \times 10^6 \text{ N/C}) \hat{e}_x$$
**Question 3(a):** Find the electric field a distance $r$ from a wire of positive charge of infinite length and constant charge per unit length $\lambda$. Also draw the diagram for the positively charged rod.

\[ \vec{A}_{cs} \text{ corresponds to area of curved surface!} \]

\[ \Phi_{total} = \Phi_{cs} + \Phi_{top} + \Phi_{bottom} \]

\[ = \vec{E} \cdot \vec{A}_{cs} + \vec{E} \cdot \vec{A}_{top} + \vec{E} \cdot \vec{A}_{bottom} \]

\[ = EA_{cs} \cos(\theta) + EA_{top} \cos(90^\circ) + EA_{bottom} \cos(90^\circ) \]

\[ = EA_{cs} + 0 + 0 \]

\[ \Phi_{inside} = E(2\pi r L) \]

\[ Q_{inside} = \lambda L \]

Using Gauss's Law

\[ \Phi = \frac{Q_{inside}}{\varepsilon_0} \]

\[ E(2\pi r L) = \frac{\lambda L}{\varepsilon_0} \]

\[ E = \frac{\lambda}{2\pi r \varepsilon_0} \]

(Magnitude of E-field at a distance $r$)

Direction is perpendicular to line of charge.

**Question 3(b):** Find the value of the electric field at a distance $r = 10\text{cm}$ from the center of a non-conducting sphere of radius $R=1\text{cm}$ which has an extra positive charge equal to $7\text{ Coulomb}$ uniformly distributed within the volume of the sphere.

If the charge at sphere (of radius $R$) is uniformly distributed, for a distance $r$ from center of sphere $R$ such that $r>R$, the E-field created by it is same as the electric field of the same amount of point charge located at the center of center of sphere. We also get the same result using Gauss's Law

\[ (E-field \ outside \ sphere) = \frac{k \ Q}{r^2} = \frac{9 \times 10^9 \times 7}{(0.1)^2} \ \text{N/C} = \frac{6.3 \times 10^{12} \ \text{N/C}} \]

\[ q = 7\text{C} \]

\[ r = 0.1\text{m} \]

Direction of E-field is out of sphere along the unit vector of sphere's surface!