CHAPTER #23: Gauss's Law

* Electric field of any charge distribution can be calculated by means of Coulomb's law.
  → But it would be difficult and tedious if the charge is distributed over a surface or volume (because then we have to perform difficult summation or integration of all small chunks).

* Fortunately, there is another method and it is dual to Coulomb's law (referred to as Gauss's law).
  → It does contain some new mathematics, which supplies an elegant shortcut for calculating the electric field, provided that the charge distribution has a high degree of symmetry.

![Electric Flux](image)

Regarded as it is proportional to the no. of field lines intercepted by the surface.
Electric flux $\Phi$ is the product of the magnitude of Electric field and the component of vector area which is perpendicular to the field.

**Mathematical Form:**

\[ \Phi_E = \varepsilon \cdot A = \varepsilon (E \cos \theta) A = \varepsilon E A \cos \theta \]

**Electric Flux** $\Phi_\varepsilon = \varepsilon A \cos \theta$. 

\[ \Phi_\varepsilon = \varepsilon A \cos \theta \]
\[ \phi_E = 3A \cos \theta = \vec{E} \cdot \vec{A} \]

\[ \vec{A} = |\vec{A}| \hat{n} \]  
(\text{normal/perpendicular unit vector})

\[ \phi_E = \int \vec{E} \cdot d\vec{A} \]

**General Arbitrary Curved Shape**

- Non-uniform electric fields
- Divide into chunks

**Note:** The lines going through across the surface from one side make a tie contributions to the flux and vice versa (from other side).

Open Surfaces:

Valid for closed surfaces (why, why?)
**Gauss' Law**

**Gaussian Surface**: Hypothetical (imaginary) closed surface enclosing the charge distribution. It can have any shape but with max level of symmetry. E.g., if the charge is spread uniformly over a sphere, we enclose the sphere with a spherical Gaussian surface.

Relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

\[ \phi = \frac{q_{\text{enc}}}{\varepsilon_0} \]

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

Relates the net flux \( \phi \) from an electric field through a closed surface (Gaussian) to the net charge enclosed by that surface.
Hold only when the net charge is located in a vacuum or air. We will modify it in later chapter (oil, glass...)

If $\text{charge} = +ve$, the net flux is outward!
If $\text{charge} = -ve$, is inward!

$S_1$: net flux
$S_2$: $-ve$ flux
$S_3$: no charge $\Rightarrow$ flux = 0

because # of lines entering is equal to # of lines leaving.

$S_4 = S_3 + S_1 + S_2$
Gauss' law and Coulomb's law

Both are different ways of describing the relation between electric charge and electric field in static equation.

By def: the area vect dA at any point is \( \perp \) to the surface & directed outward from interior.

From Symmetry of situation:

\( \vec{E} \) is also \( \perp \) to the surface and directed outward.

\[
\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}
\]

\[
\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}
\]

Our Gaussian surface is spherical (highly symmetrical) it has the same value of \( \vec{E} \) on the spherical surface.
\[ \varepsilon \int dA = \frac{q}{\varepsilon_0} \]

\[ \varepsilon (4\pi r^2) = \frac{q}{\varepsilon_0} \]

(i surface area of sphere = \(4\pi r^2\))

\[ \varepsilon = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]

\textit{Coulomb's Law}

Sample Prob: 12, 3, 4