1. An electroscope for the detection and measurement of electric charge consists of a fixed cork ball and a suspended cork ball (see Fig. 1). The mass of the suspended ball is $1.5 \times 10^{-4}$ kg, and the length of the suspension thread is 10 cm. The fixed ball is located 10 cm directly below the point of suspension of the suspended ball. Assume that when equal electric charges are placed on the two balls, the electric repulsive force pushes the suspended ball up so its thread makes an angle of 45° with the vertical. What is the magnitude of the electric charge?

Figure 1: A charge suspended by a thread and a fixed charge.

[10]
2. Four point charges of $\pm Q$ are arranged on the corners of a square of side $L$ as illustrated in below Figure. What is the net electric force that these charges exert on a point charge $q$ placed at the center of the square? \[10\]

3. A spherical shell has net charge only on its inner and outer surfaces. The total charge on the entire shell is $Q_{\text{total}} = -1.0 \times 10^{-8}$ C. The charge on the inner surface of the shell is $Q_{\text{inner}} = +2.0 \times 10^{-8}$ C. What charge is on the outer surface of the shell? \[05\]

4. Find out the expression for the electric field of a quadrupole and verify that it varies with $1/r^4$. \[10\]

5. Four closed surfaces, $S_1$ through $S_4$, together with the charges $-2Q$, $Q$, and $-Q$ are sketched in following figure:

(i) Find the electric flux through each surface.
(ii) Also sketch the electric flux corresponding to each surface. \[10+5\]

6. A particle with charge $Q$ is located immediately above the center of the flat face of a hemisphere of radius $R$ as shown in below figure. What is the electric flux through

1. the curved surface?
2. the flat face? \[5+5\]

Due Date: 24th November 2020
Solutions:

Question No. 1:

We are given that

Mass of suspended ball \( m = 1.5 \times 10^{-4} \text{ kg} \)

Length of suspension thread = \( l = 10 \text{ cm} = 0.10 \text{ m} \)

(Distance between point of suspension and fixed ball) = \( x = 10 \text{ cm} = 0.10 \text{ m} \)

(Angle between fixed and suspended ball) = \( \theta = 45^\circ \)

(Magnitude of electric charge on each ball) = \( q = ? \)

In order to calculate the charge, at first we shall calculate electric force of repulsion and distance between two charges.
From the figure we can see that the distance between two balls $h$ is

$$h = l \sin 22.5^\circ + r \sin 22.5^\circ$$

$$= (0.1 \text{m}) \sin 22.5^\circ + (0.1 \text{m}) \sin 22.5^\circ$$

$$= 0.2 \text{m} \times \sin 22.5^\circ \quad \text{(1)}$$

If suspended ball is in equilibrium, the sum of all the forces acting on it will be equal to zero. Consider forces on suspended ball along $y-$axis. We can see that weight of the ball is balanced by the $y$-component of the force and tension i.e.

$$F \sin 22.5^\circ + T \sin 45^\circ = mg$$

$$\Rightarrow F = \frac{mg - T \sin 45^\circ}{\sin 22.5^\circ} \quad \text{(2)}$$

where $F$ is the electric force of repulsion between two ball. According to Coulomb's law, this force will be...
\[ F = k \frac{Q^2}{\lambda^2} = k \frac{Q^2}{\lambda^2}. \quad (3) \]

Similarly consider force on suspended ball along \( x \)-axis

\[ F \cos 22.5^\circ = T \cos 45^\circ. \]

\[ \Rightarrow T = F \frac{\cos 22.5^\circ}{\cos 45^\circ}. \]

Substitute value of \( T \) in eq. (2)

\[ F = \frac{mg}{\sin 22.5^\circ} - F \frac{\cos 22.5^\circ}{\cos 45^\circ} \cdot \frac{\sin 45^\circ}{\sin 22.5^\circ}. \]

\[ F = \frac{mg}{\sin 22.5^\circ} - F \frac{\tan 45^\circ}{\tan 22.5^\circ}. \]

\[ F \left( 1 + \frac{\tan 45^\circ}{\tan 22.5^\circ} \right) = \frac{mg}{\sin 22.5^\circ} = \frac{1.5 \times 10^{-4} \text{ kg} \times 9.8 \text{ m/s}^2}{\sin 22.5^\circ}. \]

\[ F(3.41) = 3.84 \times 10^{-3} \]

\[ F = 1.13 \times 10^{-3} \text{ N} \]

Eq. (3) can be written as

\[ Q^2 = \frac{Fx^2}{k}. \]
\[ Q = \sqrt{\frac{F}{k}} \]

Substituting values of \( F, \lambda \) and \( k \)
we get:

\[ Q = \sqrt{\frac{1.13 \times 10^{-3} \text{N}}{9 \times 10^9 \text{Nm}^{-2}}} \times 0.2 \times \sin 22.5^\circ \]

\[ Q = \sqrt{1.253 \times 10^{-13} \text{C}^2/\text{m}^2} \times 0.076 \text{m} \]

\[ = 3.54 \times 10^{-7} \text{C} \times 0.076 \text{m} \]

\[ Q = 2.69 \times 10^{-8} \text{C} \]

\[ Q = 2.7 \times 10^{-8} \text{C}. \]

Hence charge on each ball will be
\[ 2.7 \times 10^{-8} \text{C}. \]
Question No. 2:

Due to the symmetry of the situation, the vertical components are equal in magnitude, but oppositely directed. Thus they cancel. The $x$ components of each of these forces is directed parallel to the positive $x$ axis as shown in the figure.

The force on the test charge due to any one of charge is:

\[ F_x = \frac{1}{4\pi\varepsilon_0} \left( \frac{Qq}{L\sqrt{2}} \right)^2 \cos \theta \quad (1) \]

Whereas,

\[ \cos \theta = \frac{\frac{L}{2}}{L\sqrt{2}/2} = \frac{\sqrt{2}}{2} \quad (2) \]

Thus Eq. 1 becomes:

\[ F_x = \frac{1}{4\pi\varepsilon_0} \left( \frac{Qq}{L\sqrt{2}} \right)^2 \frac{\sqrt{2}}{2} \quad (3) \]

By superposition principle of electric forces, the net force along $x$ axis on the test charge is given by:

\[ F_x = 4\sqrt{2} \frac{1}{4\pi\varepsilon_0} \frac{Qq}{L^2} \quad (4) \]

\[ F_x = 4\sqrt{2} \frac{1}{4\pi\varepsilon_0} \frac{Qq}{L^2} \quad (5) \]

Therefore, the net force on the charge $+q$ is

\[ \vec{F} = 4\sqrt{2} \frac{1}{4\pi\varepsilon_0} \frac{Qq}{L^2} \hat{i} \quad (6) \]
Question No. 3:

\[ Q_{outer} = Q_{total} - Q_{inner} = (-1.0 \times 10^{-8} \text{C}) - (+2.0 \times 10^{-8} \text{C}) = -3.0 \times 10^{-8} \text{C} \]  \hspace{1cm} (7)

Question No. 4:

The electric quadrupole

It consists of two dipoles of equal magnitude placed close to each other and in opposite directions.

**EXRESS** Consider the point \( P \) on the axis, a distance \( z \) to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is given by \( qd/2\pi\varepsilon_0(z - d/2)^2 \) while the field produced by the left dipole is \( -qd/2\pi\varepsilon_0(z + d/2)^2 \).

**ANALYZE** Use the binomial expansions

\[
(z - d/2)^3 = z^3 - 3z^2(z - d/2)
\]

\[
(z + d/2)^3 = z^3 - 3z^2(z + d/2)
\]

We obtain

\[
E = \frac{qd}{2\pi\varepsilon_0(z - d/2)^2} = \frac{qd}{2\pi\varepsilon_0(z + d/2)^2} \approx \frac{qd}{2\pi\varepsilon_0} \left[ \frac{1}{z^2} + \frac{3d}{2z^4} \right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}.
\]

Since the quadrupole moment is \( Q = 2qd^2 \), we have \( E = \frac{3Q}{4\pi\varepsilon_0 z^4} \).

Question No. 5:

\[ \Phi_E = \frac{q_{in}}{\varepsilon_0} \]

Through \( S_1 \)

\[ \Phi_E = \frac{-2Q + Q}{\varepsilon_0} = \frac{-Q}{\varepsilon_0} \]

Through \( S_2 \)

\[ \Phi_E = \frac{+Q - Q}{\varepsilon_0} = 0 \]

Through \( S_3 \)

\[ \Phi_E = \frac{-2Q + Q - Q}{\varepsilon_0} = \frac{-2Q}{\varepsilon_0} \]

Through \( S_4 \)

\[ \Phi_E = 0 \]
Question No. 6:

(a) With $\delta$ very small, all points on the hemisphere are nearly at a distance $R$ from the charge, so the field everywhere on the curved surface is $\frac{kQ}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

$$\Phi_{\text{curved}} = \int \mathbf{E} \cdot d\mathbf{A} = E_{\text{local}} A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(\frac{kQ}{R^2}\right) \left(\frac{1}{2} \frac{4\pi R^2}{4\pi} \right) = \frac{1}{4\pi \varepsilon_0} Q(2\pi) = \frac{+Q}{2\varepsilon_0}$$

(b) The closed surface encloses zero charge so Gauss’s law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \quad \text{or} \quad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = -\frac{Q}{2\varepsilon_0}$$