Simple Harmonic Motion and Uniform Circular Motion

1. Galileo's Story (1610 year, four principal moons of Jupiter...)

   Manifestation of SHM. But how moon can move back and forth?

   Latest technology tells us that SHM is the projection of circular motion on a diameter of the circle in which the circular motion occurs.

Example

\[ y = A \sin(\omega t + \phi) \]

- Satellite
- Particle

\[ \phi = 0 \]

\[ \phi \neq 0 \]
For particle perspective: \[ \chi = A \cos(\omega t + \phi) \]

While for satellite perspective; the satellite moves around the circle with angular position:
\[ \Theta = \omega t; \quad \phi = 0 \]

\( \chi \)-component of satellite: \[ \chi_{\text{sat}} = A \cos \Theta \]

\( \chi_{\text{sat}} = A \cos(\omega t) \)

In general: \[ \chi_{\text{sat}} = A \cos(\omega t + \phi) \]
Velocity Graph:

$V_p$ direction is inward due to its expression:

$$V_p = -\omega A \sin(\omega t + \phi)$$

Acceleration Graph:

$$a_p = -\omega^2 A \cos(\omega t + \phi)$$

Video Clip
Damped Simple Harmonic Motion

Damped Oscillations: When the motion of an oscillator is reduced by an external force.

Working:
- A broad blade which pushes or is pushed by wind
- As the vane moves up and down, liquid exerts drag force

\[ F_d = -bv \]

Equation of Motion

\[ F_s = -kx \]
\[ F_d = -bv \]

Newton's 2nd Law:

\[ F_{net,x} = ma \]
\[ -bv - kx = ma \]
\[ ma + bv + kx = 0 \]
\[ m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = 0 \]

2nd Order differential eqn. and its solution is

\[ x(t) = A \cos(\omega' t + \phi) e^{-bt/2m} \]

damped frequency

\[ \omega' = \sqrt{\frac{k}{m} \frac{-b^2}{4m^2}} \]

discuss it!

\[ b=0 \Rightarrow \omega' = \omega = \sqrt{\frac{k}{m}} \]
Oscillations

Free Oscillations

"Oscillation without any pushing called free oscillation."

Forced/Driven Oscillations

"If someone pushes the swing periodically then it is called forced oscillation."

Angular Frequencies

Natural Angular Frequency

The angular frequency at which it would oscillate if it were suddenly disturbed and left to oscillate freely.

Or

The angular frequency of a free oscillation.

Driven Angular Frequency

The angular frequency caused by the driven oscillations.

The angular frequency of the external driving force causing the driven oscillations.

\[ x(t) = A \cos(\omega_0 t + \phi) \]
A phenomenon occurs only when the frequency at which an external force is periodically applied is equal or nearly equal to one of the natural frequencies of the system on which it acts.

**Resonance**

\[ \omega_d = \omega \]

Phenomenon in which a vibrating system or external force oscillate with greater amplitude at specific frequencies.

E.g., if you push a swing at its natural frequency, the displacement and velocity amplitudes will increase to large values.

**What happens if you push the swing with higher frequency?** (lower amplitudes)

![Amplitude vs. \( \frac{\omega_d}{\omega} \)]

1. \( b = 140 \text{ g/s} \)
2. \( b = 70 \text{ g/s} \)
3. \( b = 50 \text{ g/s} \)