**Simple Harmonic Oscillator**

Consists of a particle coupled to an ideal, massless spring that obeys Hooke's law.

"A spring that provides a force is proportional to the elongation or compression of the spring." \( F = -kx \)

**Importance of SHO**

- Many physical systems are mathematically equivalent to SHO \( \Rightarrow \) these systems have same equation of motion as SHO's.

  E.g., A pendulum, a tuning fork, atoms in a diatomic molecule etc.

Mathematically, Hooke's Law: \( F = -kx \)

To find \( k \)?

According to Newton's 2nd Law: \( F = ma \)

\( F = m(-\omega^2x) \)
\[-kx = -mw^2x \Rightarrow K = mw^2\]

\[\Rightarrow \omega = \sqrt{\frac{K}{m}}\]

As we know \(T = \frac{2\pi}{\omega}\), \(\omega = \frac{2\pi}{T}\)

\[\omega = 2\pi f = \frac{2\pi}{T}\]

\[T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{K}}\]

Results:

- Frequency of oscillation is unaffected by the amplitude with which it has been in motion.

- The oscillator has a frequency of 2 Hz when oscillating with small amplitude. Then it is also has a frequency of 2 Hz with large amplitude.
Energy in Simple Harmonic Motion

According to law of conservation of energy:

\[ E = K \cdot E + U \]

- Kinetic energy of the particle moving is:
  \[ K \cdot E = \frac{1}{2} m v^2 \]

- Potential energy of a particle performing SHM is:
  \[ U = \frac{1}{2} k x^2 \]

Calculation
As we know that the change \( \Delta U \) in gravitational potential energy is defined by the negative of the work done on the object by gravitational force:

\[ \Delta U = -W \]

\[ W = \int_{x_i}^{x_f} F(x) \, dx \]

\[ (iW = Fd) \]

\[ \Delta U = - \int_{x_i}^{x_f} (-R x) \, dx = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \]
\( \Delta U = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \)

We chose relaxed position \((x_i=0)\) as a reference frame. So,

\[
U = \frac{1}{2} k x^2
\]
The motion of a particle: \[ x(t) = A \cos(\omega t + \delta) \]

\[ v(t) = -A \omega \sin(\omega t + \delta) \]

\[ U(t) = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \delta) \]

while \[ K \cdot E = \frac{1}{2} m v^2 = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \delta) \]

Since \[ k = m \omega^2 \]

\[ K \cdot E = \frac{1}{2} k A^2 \sin^2(\omega t + \delta) \]

\[ E = K + U = \frac{1}{2} k A^2 \]

\[ E = \frac{1}{2} k A^2 \]

The energy of the motion is constant and is proportional to the square of the amplitude of the oscillation.
Angular S.H.M.

- Take a suspension wire whose twisting is associated with the springiness or elasticity rather than the extension or compression of a spring.
- Attach a disk to it; one end of a wire is tied to one end.

This kind of setup is called torsion pendulum.

$\theta_0$ $\theta_0$

$\theta_0$

Angular amplitude

Angular version of linear S.H.O.

If we rotate the disk by some angular displacement from its rest position ($\theta = 0$),
Rotating the disk through an angle $\theta$ in either direction introduces a restoring torque given by:

$$\tau = -K \theta$$

Angular form of Hooke's Law:

Angular equivalence:

$$T = 2\pi \sqrt{m/R} = \frac{2\pi}{\sqrt{K}}$$
Simple Pendulum

- Bob of mass ‘m’ suspended by
- a string (massless) or a rod from some fixed point.

We will reckon this angle $\theta$ as the on the right side of the vertical and -ve on the left side.

When we release the ball from point A, the motion can be regarded as rotation about a horizontal axis through the point of suspension.

The $\text{EOM}$ is:

$$\alpha I = 2 ma$$

Free-body diagram

- Suspension force or tension
- Angular acceleration
- $\omega = mg$
\( F_g = mg \sin \theta \)

Only tangential component produces torque.

\[ \Rightarrow \text{Restoring torque: } 2 = -rF_g \]

\[ 2 = -I \frac{d^2 \theta}{dt^2} \quad (2) \]

From eq. (1) and (2):

\[ I \frac{d^2 \theta}{dt^2} = -mg \sin \theta \]

For small angles: \( \sin \theta \approx \theta \)

\[ \Rightarrow I \frac{d^2 \theta}{dt^2} = -mg \theta \]

\[ \alpha = -\frac{mg \theta}{I} \]

\[ \Rightarrow \alpha = -\theta \quad (3) \]

\[ \text{v.u.v. Inf. Result.} \]

Discussion

In Linear (1D) SHM: \( \alpha(t) = -\omega^2 x(t) \)

\[ \Rightarrow \alpha = -x \]

In Simple Pendulum (2D) SHM: \( \alpha \approx -\theta \)

\[ \alpha(t) = -\omega^2 x(t) \approx [\alpha(t) = -\omega \theta(t)] \]

Angular equivalence property
\[ \omega^2 = \frac{mgI}{I} \]  

From equation (32.4) we know that  

\[ \omega = \frac{2\pi}{T} \]  

\[ \sqrt{\frac{mgl}{I}} = \frac{2\pi}{T} \]  

\[ T = 2\pi \sqrt{\frac{I}{mgI}} \]  

\[ I = ml^2 \quad \text{Rotational inertia of the string-bob system} \]  

\[ T = 2\pi \sqrt{\frac{l}{g}} \]